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Aspects of open-closed duality in a background B-field, II

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Abstract: It was shown in [13], in the context of bosonic theory that the IR singular terms that arise as a result of integrating out high momentum modes in nonplanar diagrams of noncommutative gauge theory can be recovered from low lying tree-level closed string exchanges. This follows as a natural consequence of world-sheet open-closed string duality. Here using the same setup we study the phenomenon for noncommutative $\mathcal{N} = 2$ gauge theory realised on a D_3 fractional brane localised at the fixed point of $\mathbb{C}^2/\mathbb{Z}_2$. The IR singularities from the massless closed string exchanges are exactly equal to those coming from one-loop gauge theory. This is as a result of cancellation of all contributions from the massive modes.

Keywords: Superstrings and Heterotic Strings, D-branes, String Duality, Non-Commutative Geometry.

Contents

1. Introduction

World sheet duality is a very fundamental feature of string theories and one expects that because of this there is a duality between open string theories and closed string theories. AdS/CFT [1] can be viewed as a particular realization of this where to leading order we see a relation between the massless sectors of the two theories. It is useful to study such dualities in other backgrounds to further elucidate the key ingredients. One such background is the constant B-field. As will be shown below because of the regulatory nature of the B-field duality statements in some cases can be made more sharply.

Open string dynamics in constant background B-field have been studied for the past few years with renewed interest for other reasons also. This is mainly due to the fact that the low energy dynamics of open strings in background constant B-field can be studied as a gauge theory on noncommutative spacetimes [2 − 5]. Field theories on noncommutative spaces are interesting in themselves, however their embedding in string theory helps us to analyse them from a wider perspective.

In the recent years there has been an extensive study of noncommutative field theories from two directions [6]. One by starting from noncommutativity at the field theory level and exploring various phenomena that arise which may be absent in commutative models. The other direction is by studying them from the point of view of string theory. Most often the latter perspective has led to better understanding of the various unusual features in the noncommutative models.

It is now well known that a generic feature of noncommutative field theories is the appearance of infrared singularities by integrating out high momentum modes propagating in the loop, popularly known as UV/IR mixing. Various attempts have been made to interpret these results in the usual wilsonian renormalisation group picture. It is shown that this phenomenon in noncommutative field theories fits into the usual notion of wilsonian

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renormalisation group if we include an infrared cutoff [8]. A different approach to cure the problem of UV/IR mixing has also been pursued [9]. It is however, not clear whether this is to be viewed as an inequivalent quantisation and therefore a different theory or a different cure to the infrared divergences.

However, the coupling of the ultraviolet to the infrared is inherent in string theory and manifests itself as a consequence of open-closed string duality. With this hindsight it was proposed that the new IR singularities should appear at the field theory level by integrating out additional massless closed string modes that couple to the gauge theory [7, 10]. This was studied for the one loop N-point tachyonic amplitude in bosonic theory in [11]. Interesting connections between the closed string tachyons and noncommutative divergences was shown in [12]. Usually the ultraviolet divergences of the open string modes can be interpreted as infrared divergences from massless closed string exchanges. In the presence of the background B-field these divergences are regulated and thus a quantitative analysis can be made. The one-loop two point diagram for open strings is a cylinder with a modular parameter t and vertex operator insertions at the boundaries. The two point one-loop noncommutative field theory diagram results in the Seiberg-Witten limit by keeping surviving terms in the integrand for the integral over t for $t \to \infty$. This limit suppresses all contributions from massive modes in the loop. The resulting diagram is that of the gauge theory with massless propagating modes. This amplitude is usually divergent in the ultraviolet when integrated over t. The source of ultraviolet divergence is the same as that of those in string theory i.e. $t \to 0$. It is therefore natural to analyse the amplitude directly in this limit when only the low lying closed string exchanges contribute. We have analysed this in [13]. We showed, in the context of bosonic string theory, that using open-closed string duality, one can recover the infrared divergent behaviour coming from nonplanar loop amplitudes in gauge theory from lowest lying tachyonic and massless tree-level closed string exchanges. However the correspondence could not be made exact as contributions from massive states in the bosonic theory are also relevant. Though the nature of the singularities matches with appropriate tensor structures, the coefficients are not equal. We argued that this equality can exactly occur in some special supersymmetric models. Open strings on fractional branes localised at the fixed point of $\mathbb{C}^2/\mathbb{Z}_2$ naturally satisfy these conditions. In this paper we study this issue in the setup of [13] in the orbifold background.

The fact that the ultraviolet behaviour of the one loop gauge theory is same as that of the massless closed string tree-level exchanges in this model with $\mathcal{N} = 2$ supersymmetry have been pointed out in [15]. Further studies as the consequence of this duality have been done by various authors [16, 17]. From the point of view of open-closed string duality this was analysed in [18]. See [19] for a recent review and references therein. We show that the UV/IR mixing phenomenon of $\mathcal{N} = 2$ gauge theory can be naturally interpreted as a consequence of open-closed string duality in the presence of background B-field. The effective action for the full two point function from gauge theory differs from that with closed string exchanges only by finite derivative corrections. However as far as the divergent UV/IR mixing term is concerned, it is exactly equal to the infrared contribution from the massless closed string exchanges. Using world-sheet duality these modes can be identified as coming from the twisted NS-NS and R-R sectors. This model was also studied in [14] in the context of closed string realization of the IR singular terms in gauge theory. Here in this paper we show how only the twisted NS-NS and R-R sectors closed strings couplings to the gauge theory survive and that the closed string interpretation of the UV/IR-terms naturally follows as a consequence of open-closed duality. The crucial feature that plays a role here is that the contributions from the massive modes cancel in this model.

This paper is organised as follows. In section 2 we give a brief review of open strings in background constant B-field and include the ingredients necessary for the computation of two point one loop open string amplitude. In section 3 we give a short review of strings on $\mathbb{C}^2/\mathbb{Z}_2$ orbifold and the massless spectrum of open strings ending on fractional D_3-brane localised at the fixed point. In section 4 we compute the two point function for one loop open strings and analyse it in the open and closed string channels. By taking the field theory limit, we show using open-closed string duality that the new IR divergent term from the nonplanar amplitude is exactly equal to the IR divergent contributions from massless closed string exchanges. We conclude this paper with discussions in the appendix A.

Conventions: we will use capital letters $(M, N, \dots)$ to denote general spacetime indices and small letters $(i, j, \dots)$ for coordinates along the D-brane.

2. Open superstring in Background B-field

In this section we begin with some preliminaries of open string dynamics in background B-field that gives rise to noncommutative gauge theory on the D_p-brane in the Seiberg-Witten limit [5]. The world-sheet action for the bosons coupled to a constant B-field is given by,

$$S_B = -\frac{1}{4\pi\alpha'}\int_\Sigma g_{MN}\partial_a X^M \partial^a X^N + \frac{1}{2}\int_{\partial\Sigma} B_{MN} X^M \partial_\tau X^N . \tag{2.1}$$

The B-field is turned on only along the world volume directions of the D_p-brane, such that, $B_{MN} \neq 0$ only for $M, N \leq p+1$ and $g_{MN} = 0$ for $M \leq p+1, N > p$. The variation of the action gives the following boundary equations for the world-sheet bosons,

$$g_{MN}\partial_\sigma X^N + 2\pi\alpha' B_{MN}\partial_\tau X^N = 0 \,|_{\sigma=0,\pi} \ . \tag{2.2}$$

The world-sheet propagator on the boundary of a disc satisfying this boundary condition is given by,

$$\mathcal{G}(y, y') = -\alpha' G^{MN} \ln\left(y - y'\right)^2 + \frac{i}{2}\theta^{MN}\epsilon\left(y - y'\right), \tag{2.3}$$

where, $\epsilon(\Delta y)$ is 1 for $\Delta y > 0$ and -1 for $\Delta y < 0$. G_{MN}, θ_{MN} are given by,

$$\begin{aligned}
G^{MN} &= \left(\frac{1}{g + 2\pi\alpha' B} g \frac{1}{g - 2\pi\alpha' B}\right)^{MN} \\
G_{MN} &= g_{MN} - (2\pi\alpha')^2 (Bg^{-1}B)_{MN} \\
\theta^{MN} &= -(2\pi\alpha')^2 \left(\frac{1}{g + 2\pi\alpha' B} B \frac{1}{g - 2\pi\alpha' B}\right)^{MN} .
\end{aligned} \tag{2.4}$$

The important point to note in the above relations is the difference between the open string metric, G and the closed string metric g. A nontrivial low energy theory results from the following limits,

$$\alpha' \sim \epsilon^{1/2} \to 0\,; \qquad g_{ij} \sim \epsilon \to 0\,, \tag{2.5}$$

where, i, j are the directions along the brane. This is the Seiberg Witten (SW) limit which gives rise to noncommutative field theory on the brane. The tree-level action for the low energy effective field theory on the brane has the following form,

$$S_{\rm YM} = -\frac{1}{g^2_{\rm YM}} \int \sqrt{G} G^{kl} G^{k'l'}\, {\rm Tr}(\hat{F}_{kl} * \hat{F}_{k'l'})\,, \tag{2.6}$$

where the $*$-product is defined by,

$$f * g(x) = e^{\frac{i}{2}\theta^{ij}\partial^y_i \partial^z_j} f(y) g(z)\mid_{y=z=x} \tag{2.7}$$

$\hat{F}_{kl}$ is the noncommutative field strength, and is related to the ordinary field strength, F_{kl} by,

$$\hat{F}_{kl} = F_{kl} + \theta^{ij}(F_{ki}F_{lj} - A_i \partial_j F_{kl}) + \mathcal{O}(F^3) \tag{2.8}$$

and,

$$\hat{F}_{kl} = \partial_k \hat{A}_l - \partial_l \hat{A}_k - i\hat{A}_k * \hat{A}_l + i\hat{A}_l * \hat{A}_k\,. \tag{2.9}$$

We now include world-sheet fermions. The action for the fermions coupled to B-field is given by,

$$S_F = \frac{i}{4\pi\alpha'} \int_\Sigma g_{MN} \bar{\psi}^M \rho^\alpha \partial_\alpha \psi^N - \frac{i}{4} \int_{\partial\Sigma} B_{MN} \bar{\psi}^N \rho^0 \psi^M\,. \tag{2.10}$$

The full action including the bosons and the fermions with the bulk and the boundary terms are invariant under the following supersymmetry transformations,

$$\begin{aligned} \delta X^M &= \bar{\epsilon}\psi^M \\ \delta\psi^M &= -i\rho^\alpha \partial_\alpha X^M \epsilon\,. \end{aligned} \tag{2.11}$$

We now write down the boundary equations by varying (2.10) with the following constraints,

$$\delta\psi^M_L = \delta\psi^M_R \mid_{\sigma=\pi} \qquad \text{and} \qquad \delta\psi^M_L = -(-1)^a \delta\psi^M_R \mid_{\sigma=0}\,, \tag{2.12}$$

where, $a = 0, 1$ gives the NS and the R sectors respectively This gives the following boundary equations,

$$g_{MN}(\psi^N_L - \psi^N_R) + 2\pi\alpha' B_{MN}(\psi^N_L + \psi^N_R) = 0\mid_{\sigma=\pi} \tag{2.13}$$

$$g_{MN}(\psi^M_L + (-1)^a \psi^M_R) + 2\pi\alpha' B_{MN}(\psi^N_L - (-1)^a \psi^N_R) = 0\mid_{\sigma=0}\ . \tag{2.14}$$

To write down the correlator for the fermions, first define,

$$\begin{aligned} \psi^M &= \psi^M_L(\sigma,\tau) \qquad\qquad 0 \le \sigma \le \pi \\ &= \left(\frac{g - 2\pi\alpha' B}{g + 2\pi\alpha' B}\right)^M_{\ N} \psi^N_R(2\pi - \sigma, \tau) \qquad\qquad \pi \le \sigma \le 2\pi\,. \end{aligned} \tag{2.15}$$

This is the usual doubling trick that ensures the boundary conditions (2.13). In the following section we would compute the two point function for the gauge field on the brane by inserting two vertex operators at the boundaries of the cylinder. Restricting ourselves to the directions along the brane, this vertex operator for the gauge field in the zero picture is given by,

$$V(p,x,y) = \frac{g_o}{(2\alpha')^{1/2}}\epsilon_j\left(i\partial_y X^j + 4p.\Psi\Psi^j\right)e^{ip.X}(x,y)\,, \tag{2.16}$$

where Ψ^i is given by,

$$\begin{aligned}\Psi^i(0,\tau) &= \frac{1}{2}\left(\psi^i_L(0,\tau) + (-1)^{a+1}\psi^i_R(0,\tau)\right) = \left(\frac{1}{g-2\pi\alpha' B}g\right)^i_j \psi^j_L(0,\tau)\\ \Psi^i(\pi,\tau) &= \frac{1}{2}\left(\psi^i_L(\pi,\tau) + \psi^i_R(\pi,\tau)\right) = \left(\frac{1}{g-2\pi\alpha' B}g\right)^i_j \psi^j_L(\pi,\tau).\end{aligned} \tag{2.17}$$

Using (2.17), the correlation function for Ψ is given by,

$$\left\langle \Psi^i(w)\Psi^j(w')\right\rangle = G^{ij}\mathcal{G}\left[{}^{\alpha}_{\beta}\right](w-w')\,, \tag{2.18}$$

where G^{ij} is the open string metric defined in (2.4) and, $\mathcal{G}\left[{}^{\alpha}_{\beta}\right](w-w')$ is given by [20],

$$\mathcal{G}\left[{}^{\alpha}_{\beta}\right](w-w') = \frac{\alpha'}{4\pi}\frac{\vartheta\left[{}^{\alpha}_{\beta}\right]\left(\frac{w-w'}{2\pi},it\right)\vartheta'\left[{}^{1/2}_{1/2}\right](0,it)}{\vartheta\left[{}^{1/2}_{1/2}\right]\left(\frac{w-w'}{2\pi},it\right)\vartheta\left[{}^{\alpha}_{\beta}\right](0,it)} \tag{2.19}$$

α, β denotes the spin structures. $\alpha = (0, 1/2)$ are the NS and the R sectors and $\beta = (0, 1/2)$ stands for the absence or the presence of the world-sheet fermion number $(-1)^F$ with ψ being antiperiodic or periodic along the τ direction on the world-sheet. w, w' are located at the boundaries of the world-sheet for the open string which is a cylinder, i.e. at $\sigma = 0, \pi$.

2.1 Open strings on $\mathbb{C}^2/\mathbb{Z}_2$ orbifold and Fractional Branes

An efficient and simple way to break $\mathcal{N} = 4$ supersymmetry and obtain gauge theories with less supersymmetries is by orbifolding the background space. Specifically strings on $\mathbb{C}^2/\mathbb{Z}_2$ gives rise to $\mathcal{N} = 2$ supersymmetric gauge theory on D3-branes with world volume directions transverse to the orbifolded planes. We will take the orbifolded directions to be $6, 7, 8, 9$ with $\mathbb{Z}_2 = \{g_i \mid e, g\}$ such that $g^2 = e$ The action of g on these coordinates is given by,

$$gX^I = -X^I \qquad \text{for} \qquad I = 6,7,8,9\,. \tag{2.20}$$

In order to preserve world sheet supersymmetry we must consider the action of $\mathbb{Z}_2$ on the fermionic partners, ψ^I. On a particular state the orbifold action is on the oscillators, ψ^I_{-r} along with the Chan-Paton indices associated with it. Let us consider the massless bosonic states from the NS sector.

$$g|i,j,\psi^I_{-1/2}\rangle = \gamma_{ii'}|i',j',\hat{g}\psi^I_{-1/2}\rangle\gamma^{-1}_{j'j}\,, \tag{2.21}$$

where γ is a representation of $\mathbb{Z}_2$ The spectrum is obtained by keeping the states that are invariant under the above action. To derive this it is easier to work in the basis where,

$$\gamma = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{2.22}$$

The action on the Chan-Paton indices can be thought of as,

$$\gamma \begin{pmatrix} 11 & 12 \\ 21 & 22 \end{pmatrix} \gamma^{-1} = \begin{pmatrix} 11 & -12 \\ -21 & 22 \end{pmatrix} . \tag{2.23}$$

Thus the diagonal ones survive for the $\mathbb{Z}_2$ action on the oscillators is $\hat{g}|\psi^I_{-1/2}\rangle = |\psi^I_{-1/2}\rangle$, i,e. for $I = 2, 3, 4, 5$ and the off diagonal ones are preserved for oscillators that are odd under the $\mathbb{Z}_2$ action, $\hat{g}|\psi^I_{-1/2}\rangle = -|\psi^I_{-1/2}\rangle$, i,e. for $I = 6, 7, 8, 9$. The spectrum can thus be summarised as,

$$A^I \longrightarrow \begin{bmatrix} A^I_1 \; A^I_2 & 2 \text{ gauge fields} & I = 2, 3 \\ \phi^I_1 \; \phi^I_2 & 4 \text{ real scalars} & I = 4, 5 \\ \Phi^I_1 \; \Phi^I_2 & 8 \text{ real scalars} & I = 6, 7, 8, 9 \end{bmatrix} . \tag{2.24}$$

The above fields can be grouped into two vector multiplets and two hypermultiplets of $\mathcal{N} = 2$ with gauge group $\mathrm{U}(1) \times \mathrm{U}(1)$. The beta function for the gauge couplings for this theory vanishes and the theory is conformally invariant. Now consider an irreducible representation $\gamma = \pm 1$. This acts trivially on the Chan-Paton indices. There is no image for the $D3$ brane. Following the above analysis we see that the spectrum consists of a single gauge field and two scalars completing the vector multiplet of $\mathcal{N} = 2$ with gauge group $\mathrm{U}(1)$. The beta function for this theory is nonzero. With a constant B-field turned on along the world volume directions of the D_3-brane,$(0, 1, 2, 3)$, the low energy dynamics on the brane will be described be noncommutative gauge theory in the Seiberg-Witten limit. In the following section we will study the ultraviolet behaviour of this theory and see how the UV divergences have a natural interpretation in terms of IR divergences due to massless closed string modes as a result of open-closed string duality.

3. Two point amplitude

In this section we compute the two point function for the gauge fields on the brane. The necessary ingredients are given in section 2 and in the appendix. The vacuum amplitude without any vertex operator insertion vanishes as a result of supersymmetry, i.e.

$$\det(g + 2\pi\alpha' B) \int_0^\infty \frac{dt}{4t} (8\pi^2\alpha' t)^{-2} \sum_{(\alpha,\beta,g_i)} Z \left[{}^{\alpha}_{\beta}\right]_{g_i} = 0 \, . \tag{3.1}$$

The factor of $\det(g+2\pi\alpha' B)$ comes from the trace over the world sheet bosonic zero modes. The sum is over the spin structures $(\alpha, \beta) = (0, 1/2)$ corresponding to the $NS - R$ sectors and the GSO projection and the orbifold projection. The elements $Z_{g_i}\left[{}^{\alpha}_{\beta}\right]$ are computed in the appendix. Let us now compute the two point function. This is given by,

$$A(p,-p) = \det(g + 2\pi\alpha' B) \int_0^\infty \frac{dt}{4t} (8\pi^2\alpha' t)^{-2} \times \\ \times \sum_{(\alpha,\beta,g_i)} Z[{}^\alpha_\beta]_{g_i} \int_0^{2\pi t} dy \int_0^{2\pi t} dy' \left\langle V(p,x,y)V(-p,x',y')\right\rangle_{(\alpha,\beta)} . \tag{3.2}$$

For the flat space, it is well known that amplitudes with less that four boson insertions vanish. However, in this model the two point amplitude survives. We will now compute this amplitude in the presence of background B-field. First note that the bosonic correlation function, $\left\langle : \partial_y X^i e^{ip.X} :: \partial_y' X^i e^{-ip.X} :\right\rangle$, does not contribute to the two point amplitude as it is independent of the spin structure. The two point function would involve the sum over the $Z_{g_i}[{}^\alpha_\beta]$ which makes this contribution zero due to (3.1). The nonzero part of the amplitude will be obtained from the fermionic part,

$$\epsilon_k\epsilon_l \left\langle : p.\Psi\Psi^k e^{ip.X} :: p.\Psi\Psi^l e^{-ip.X} :\right\rangle = \epsilon_k\epsilon_l p_i p_j \left(G^{il}G^{jk} - G^{ij}G^{kl}\right) \times \\ \times \mathcal{G}^2 [{}^\alpha_\beta] (w - w') \langle : e^{ip.X} :: e^{-ip.X} :\rangle . \tag{3.3}$$

For the planar two point amplitude, both the vertex operators would be inserted at the same end of the cylinder (i.e. at $w = 0 + iy$ or $\pi + iy$). In this case, the sum in the two point amplitude reduces to,

$$\begin{aligned}\sum_{(\alpha,\beta,g_i)} Z[{}^\alpha_\beta]_{g_i} \mathcal{G}^2 [{}^\alpha_\beta] \left(\frac{i\Delta y}{2\pi}\right) &= \sum_{(\alpha,\beta)} Z[{}^\alpha_\beta]_e \mathcal{G}^2 [{}^\alpha_\beta] (i\Delta y/2\pi) + \\ &+ \sum_{(\alpha,\beta)} Z[{}^\alpha_\beta]_g \mathcal{G}^2 [{}^\alpha_\beta] \left(\frac{i\Delta y}{2\pi}\right) \\ &= \frac{4\pi^2}{\eta(it)^6 \vartheta_1^2(i\Delta y/2\pi, it)} \sum_{(\alpha,\beta)} \vartheta^2(0,it) [{}^\alpha_\beta] \vartheta^2 [{}^\alpha_\beta] \left(\frac{i\Delta y}{2\pi}, it\right) + \\ &+ \frac{16\pi^2}{\vartheta_1^2(i\Delta y/2\pi, it)\vartheta_2^2(0,it)} \times \\ &\times \left[\vartheta_3^2\left(\frac{i\Delta y}{2\pi}, it\right)\vartheta_4^2(0,it) - \vartheta_4^2\left(\frac{i\Delta y}{2\pi}, it\right)\vartheta_3^2(0,it)\right],\end{aligned}$$

where, $\Delta y = y - y'$. We have separated the total sum as the sum over the two $\mathbb{Z}_2$ group actions. In writing this out we have used the following identity

$$\eta(it) = \left[\frac{\partial_\nu \vartheta_1(\nu, it)}{-2\pi}\right]^{1/3}_{\nu=0} . \tag{3.4}$$

Now, the first term vanishes due to the following identity

$$\sum_{(\alpha,\beta)} \vartheta [{}^\alpha_\beta] (u) \vartheta [{}^\alpha_\beta] (v) \vartheta [{}^\alpha_\beta] (w) \vartheta [{}^\alpha_\beta] (s) = \\ = 2\vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (u_1) \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (v_1) \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (w_1) \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (s_1) , \tag{3.5}$$

where,

$$u_1 = \frac{1}{2}(u+v+w+s) \qquad v_1 = \frac{1}{2}(u+v-w-s)$$
$$w_1 = \frac{1}{2}(u-v+w-s) \qquad s_1 = \frac{1}{2}(u-v-w+s) \tag{3.6}$$

and noting that, $\vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(0,it) = 0$, in the same way as the flat case that makes amplitudes with two vertex insertions vanish. The second term is a constant also due to,

$$\vartheta_4^2(z,it)\vartheta_3^2(0,it) - \vartheta_3^2(z,it)\vartheta_4^2(0,it) = \vartheta_1^2(z,it)\vartheta_2^2(0,it)\,. \tag{3.7}$$

For the nonplanar amplitude, which we are ultimately interested in, we need to put the two vertices at the two ends of the cylinder such that, $w = \pi + iy$ and $w^{'} = iy^{'}$. It can be seen that the fermionic part of the correlator is constant and independent of t, same as the planar case following from the identity (3.7). The effect of nonplanarity and the regulation of the two point function due to the background B-field is encoded in the correlation functions for the exponentials. The two point function thus reduces to,

$$A(p,-p) \sim \epsilon_k\epsilon_l p_i p_j \left(G^{il}G^{jk} - G^{ij}G^{kl}\right)\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha^{'}t)^{-2}\int_0^{2\pi t} dy dy^{'}\left\langle e^{ip.X}e^{-ip.X}\right\rangle\,. \tag{3.8}$$

The noncommutative gauge theory two point function is obtained in the limit $t\to\infty$ and $\alpha^{'}\to 0$. The correlation function in this limit can be computed from the bosonic correlation functions [21, 22]. We give below the function for the nonplanar case in this limit.

$$\begin{aligned}\left\langle e^{ip.X}e^{-ip.X}\right\rangle &= \exp\left\{-p^2t\Delta x(\Delta x-1) - \frac{1}{4t}p_i(g^{-1}-G^{-1})^{ij}p_j\right\}\\ &= \exp\left\{-p^2t\Delta x(\Delta x-1) - \frac{\tilde{p}^2}{4t}\right\},\end{aligned} \tag{3.9}$$

where, $\tilde{p} = (\theta p)$. We have redefined the world sheet coordinate as $\Delta x = \Delta y/(2\pi t)$ and have scaled $t\to t/(2\pi\alpha^{'})$. We have also used the following relation in writing down the last expression.

$$g^{-1} = G^{-1} - \frac{(\theta G\theta)}{(2\pi\alpha^{'})^2}\,. \tag{3.10}$$

The first term in the exponential in (3.9) regulates the integral over t in the infrared, for $p\neq 0$ and the second term regulates it in the ultraviolet that is usually observed in noncommutative field theories. The $t\to\infty$ limit suppresses the contributions from all the open string massive modes. However as far as the field theory divergence is concerned they still come from the $t\to 0$ region. We can thus break the integral over t into two intervals $1/\Lambda^2\alpha^{'} < t < \infty$ and $0 < t < 1/\Lambda^2\alpha^{'}$ (see figure 1). The second one which is the source of the UV divergence is also the regime dominated by massless closed string exchanges. We now evaluate the two point function in this limit. First, the correlation function for the exponential in the $t\to 0$ limit is given by

$$\left\langle e^{ip.X}e^{-ip.X}\right\rangle = \exp\left\{-\frac{\alpha^{'}\pi}{2t}p_i g^{ij}p_j\right\}, \tag{3.11}$$

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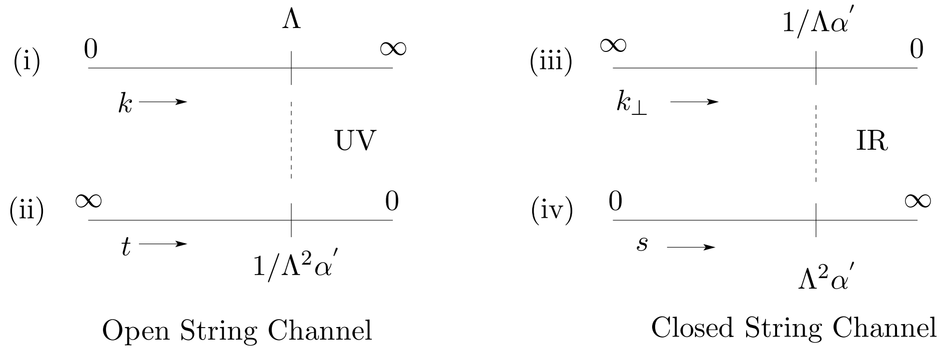

Figure 1: UV and IR regions in the open and closed string channels. (i) k is the momentum in the gauge theory one loop diagram. (ii) t is the modulus of the cylinder in the open string one loop diagram (iii) $k_\perp$ is the transverse momentum of the closed string modes emitted from the brane (iv) $s = 1/t$.

where g^{ij} is the closed string metric. Modular transformation , $(t \to 1/t)$ allows us to rewrite the one loop amplitude as the sum over closed string modes in a tree diagram. In the limit $t \to 0$, the amplitude will be dominated by massless closed string modes. In this model however, the effect of the massive modes in the loop cancel amongst themselves for any value of t. In the open string channel the $t \to 0$ limit would usually be contributed by the full tower of open string modes. However since we have seen that the effect of the massive string modes cancel anyhow for all values of t, the contribution to this limit from the open string modes comes only from the massless ones. The additional term in (3.9) as compared to (3.11) gives finite derivative corrections to the effective action. These would in general require the massive closed string states for its dual description. Without these derivative corrections, the contributions from the massless open string loop and the massless closed string tree are exactly equal. The divergent ultraviolet behaviour of the massless open string modes can thus be captured by the the massless closed string modes that have momentum in the limit $[0, 1/\Lambda\alpha^{'}]$. The amplitude can now be written as,

$$A(p,-p) = V_4 \det(g + 2\pi\alpha^{'} B) \left(\frac{g_o^2}{8\pi^2\alpha^{'}}\right) \epsilon_k \epsilon_l p_i p_j \left(G^{il}G^{jk} - G^{ij}G^{kl}\right) I(p)\,, \qquad (3.12)$$

where,

$$\begin{aligned} I(p) &= \int ds s^{-1} \exp\left\{-\frac{\alpha^{'}\pi s}{2} p_i g^{ij} p_j\right\} \\ &= 4\pi \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 + p_i g^{ij} p_j}\,. \end{aligned} \qquad (3.13)$$

The integral is written in terms of $s = 1/t$ and in the last line we have rewritten it as an integral over $k_\perp$, the momentum in the directions transverse to the brane for closed strings. The nonzero contribution to the two point amplitude in (3.2) comes from the $\mathrm{Tr}_{NS}\left[g q^{L_0}\right]$ and $\mathrm{Tr}_{NS}\left[g(-1)^F q^{L_0}\right]$, that are evaluated in (A.34). These correspond to antiperiodic (NS-NS) and periodic (R-R) closed strings in the twisted sectors respectively.

The fractional D_3-brane is localised at the fixed point of $\mathbb{C}^2/\mathbb{Z}_2$. Thus the twisted sector closed string states that couple to it are twisted in all the directions of the orbifold. These modes are localised at the fixed point and are free to move in the six directions transverse to the orbifold. This is the origin of the momentum integral (3.13) in two directions transverse to the D-brane. For the twisted sector the ground state energy for both the NS and the R sectors vanish. In the NS sector the massless modes come from ψ_0^I, $I = 6, 7, 8, 9$ oscillators which form a spinor representation of $SO(4)$. With the GSO and the orbifold projections, the closed string spectrum is given by, $2 \times 2 = [0] + [2]$. The [0] and the self-dual [2] constitute the four massless scalars in the NS-NS sector. Similarly, in the R sector, the massless modes are given by ψ_0^I for $I = 2, 3, 4, 5$. Thus giving a scalar and a two-form self-dual field in the closed string R-R sector. The couplings for the massless closed string states to the fractional D_3-brane have been worked out by various authors. See for example [17].

As we are interested in seeing the ultraviolet effect of the open string channel as an infrared effect in the closed string channel, we must cut off the s integral at the lower end at some value $\Lambda^2\alpha^{'}$ corresponding to the UV cutoff for the momentum of the massless closed strings in the directions transverse to the brane.

$$I(p, \Lambda) \sim \int_{\Lambda^2\alpha^{'}}^{\infty} \frac{ds}{s} e^{-\alpha^{'} p^2 s} \sim \int_0^{\infty} d^2 k_\perp \frac{e^{-(k_\perp^2 + p^2)\Lambda^2\alpha^{'}}}{k_\perp^2 + p^2} . \tag{3.14}$$

The integral over $k_\perp$ thus receives contributions upto $k_\perp \sim \mathcal{O}(1/\Lambda\alpha^{'})$, so that,

$$I(p, \Lambda) = 4\pi^2 \ln \left(\frac{p_i g^{ij} p_j + 1/(\Lambda\alpha^{'})^2}{p_i g^{ij} p_j} \right) . \tag{3.15}$$

This is the behaviour of the two point function for two gauge fields in $\mathcal{N} = 2$ theory. For the noncommutative theory it is regulated for $p \neq 0$. The fact that we are able to rewrite the gauge theory two point function as massless closed string tree-level exchanges is very specific to the $\mathcal{N} = 2$ theory. The computations above show that the origin of this can be traced to open-closed string duality where the orbifold background cancels all contributions from the massive states as far as the UV singular terms are concerned. The background B-field in the SW limit only acts as a physical regulator.

4. Discussions

In the previous sections we have addressed the issue of open closed duality in string theory in the presence of B fields. This duality lies at the heart of duality between gravity and gauge theory as exemplified by the AdS/CFT conjecture. In that situation the gauge theory has $N = 4$ supersymmetry and is finite. The duality of the annulus diagram then reduces to a trivial identity namely, 0=0. To get something non trivial one reduces the amount of supersymmetry by orbifolding, but taking care to preserve enough supersymmetry that there are no tachyons. In this case one loop amplitudes are divergent. One can compare divergences in the closed and open string channels and if one makes a suitable identification of the cutoffs one can show the equality of amplitudes. What we have done in this paper is to consider this theory in the presence of B-fields so that some of the amplitudes (non planar) are actually finite and can be compared in an exact way. The B field plays the role of a regulator that preserves the duality.

Once we turn on a B-field we also make contact with another phenomenon: UV/IR mixing that is known to happen in non commutative field theories. This acquires has a natural explanation when we consider this theory as the $\alpha' \to 0$ limit of a string theory a la Seiberg-Witten. The open string loop UV region is reproduced by closed string trees with small (i.e. IR) momentum exchange. The B-field acts as a regulator for both amplitudes but the regulation goes away as the external momentum goes to zero. It is not surprising that the tree diagram diverges as the external momentum goes to zero, but by the duality map this must also be true for the UV divergence of the gauge theory.

In more detail we have studied a noncommutative $\mathcal{N} = 2$ gauge theory realised on a fractional D_3-brane localised at the fixed point of $\mathbb{C}^2/\mathbb{Z}_2$ orbifold.

The one loop two point open string amplitude gives the gauge theory two point amplitude in the $\alpha' \to 0$ SW limit. This assumes there is no tachyon as is the case in the model studied here. We then see that the UV divergences of the gauge theory comes from the $t \to 0$ end that is dominated by massless closed strings. In general the massless closed string exchanges account for the UV contribution due to all the open string modes. and similarly the dual description of the gauge theory would thus require the contributions from all the massive closed string states as well. But in the supersymmetric case that is studied in this paper, the contributions from the massive modes cancel and hence the duality is between the finite number of massless states on both the open and the closed string sides. This is what is manifested through the equality of (3.15) to the gauge theory amplitude, both ends being regulated by the presence of the B-field.

To see the closed string coupling to the gauge theory, consider for the moment, the bosonic theory [13]. The one loop open string amplitude is given by,

$$\mathcal{Z} \sim \int \frac{dt}{t} (\alpha' t)^{-\frac{p+1}{2}} \eta(it)^{-(D-2)} \exp\left(-\frac{C}{\alpha' t}\right). \tag{4.1}$$

Here C is the constant independent of t and is a function of the B-field and external momenta. The expansion of the η-function in the $t \to \infty$ limit, in the the open string channel gives,

$$\mathcal{Z}_{op} \sim \int \frac{dt}{t} (\alpha' t)^{-\frac{p+1}{2}} \left[e^{2\pi t} + (D-2) + O\left(e^{-2\pi t}\right)\right] \exp\left(-\frac{C}{\alpha' t}\right). \tag{4.2}$$

Let us set $\alpha' t = T$

$$\mathcal{Z}_{op} \sim \int \frac{dT}{T} (T)^{-\frac{p+1}{2}} \left[e^{2\pi \frac{T}{\alpha'}} + (D-2) + O\left(e^{-2\pi \frac{T}{\alpha'}}\right)\right] \exp\left(-\frac{C}{T}\right). \tag{4.3}$$

The $O(1)$ term in the expansion corresponds to the massless open string modes in the loop. If we take the $\alpha' \to 0$ limit the contribution of the massive modes drop out. If we ignore the tachyon we get the massless mode contribution. In the supersymmetric case there is no tachyon. However in the present case dropping the tachyon term makes an exact comparison of the massless sectors of the two cases meaningless because the powers of α' cannot match. Nevertheless the comparison is instructive.

The UV contribution of (4.2), as shown in figure 1, comes from the region $0 < t < 1/\Lambda^2\alpha^{'}$. The UV divergences coming from this region is regulated by C. In the closed string channel we have,

$$\begin{aligned}\mathcal{Z}_{cl} &\sim \int ds(\alpha^{'})^{-\frac{p+1}{2}} s^{-l/2} \left[e^{2\pi s} + (D-2) + O(e^{-2\pi s})\right] \exp\left(-\frac{Cs}{\alpha^{'}}\right) \\ &\sim (\alpha^{'})^{-\frac{p+1}{2}} (\alpha^{'})^{\frac{l}{2}-1} \int d^l k_\perp \frac{1}{k_\perp^2 + C/\alpha'^2} \,. \end{aligned} \tag{4.4}$$

The $\alpha' \to 0$ limit does not pull out the massless sector (even if we ignore the tachyon) and this makes it clear that in general all the massive closed string modes are required to reproduce the massless open string contribution. But let us focus on the massless states of the closed string sector. In the second expression of (4.4), we have kept only the contribution from the massless closed string mode. This expression can be interpreted as the amplitude of emission and absorption of a closed string state from the D_p-brane with transverse momentum $k_\perp$, integrated over $0 < k_\perp < 1/\Lambda\alpha^{'}$. The domain of the $k_\perp$ integral corresponds to the UV region in the open string channel. $l = D - (p+1)$, is the number of transverse directions in which the closed string propagates. For $l \neq 2$ there is an extra factor of $(\alpha^{'})^{\frac{l}{2}-1}$ in (4.4) over (4.2), that makes the couplings of the individual closed string modes vanish when compared to the open string channel. It is only for the special case $l = 2$ that the powers match.

In the supersymmetric case, for the $\mathbb{C}^2/\mathbb{Z}_2$ orbifold, we have seen that the closed strings that contribute to the dual description of the nonplanar divergences are from the twisted sectors. They are free to move in 6 directions transverse to the orbifold. For the D_3-brane that is localised at the fixed point with world volume directions perpendicular to the orbifold, these closed string twisted states propagate in exactly two directions transverse to the brane. Thus in this case $l = 2$ and from the above discussions this makes the power of $\alpha^{'}$ in the coupling of closed string with the gauge field strength same as that of the open string channel.

Although in general, the closed string couplings to the gauge field when the closed string modes are restricted to the massless ones do not give the same normalisation as the gauge theory, the massive closed string modes are expected to contribute so that the normalisations at both the ends are equal. This is guaranteed by open-closed string duality. For the $\mathbb{C}^2/\mathbb{Z}_2$ orbifold, since the massive states cancel, the finite number of closed string modes must give the same normalisation as the gauge theory two point function. This is the reason why we are able to see the IR behaviour of noncommutative $\mathcal{N} = 2$ theory in terms of only the massless closed string modes in the twisted sectors.

The role played by the B-field is essentially that of a regulator that preserves the open closed duality. The fact that we see the UV divergence at the field theory level as IR divergence depends on the the special nature of this regulator that is dependent on the B-field and external momenta, thus giving rise to UV/IR mixing in noncommutative gauge theories. However the B-field does not affect the correspondence between the modes on the open and closed string sides that arise as a result of the world-sheet duality. The only modification of the partition function due to the B-field is the inclusion of a constant determinant (see the appendix). In conclusion, the IR divergences in noncommutative gauge theories that arise by integrating over high momentum modes in the loops can be seen as IR divergences due to closed string exchanges, as a result of open-closed string duality. The question of whether a finite or infinite number of closed string modes are necessary for the dual description depends on the commutative theory without the B-field.

At higher orders one expects the duality to be true for the full string theory and not for the massless sectors. But in limits such as in the AdS/CFT case one can expect a duality for the massless sectors. It would be interesting to study the corresponding AdS/CFT — like limit here. This is presumably some orbifolded version of the AdS/CFT [24] with B-field [25].

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A. Evaluation of vacuum amplitude

In this appendix we calculate the vacuum amplitude for the open strings with end points on a D_3-brane that is located at the fixed point of $\mathbb{C}^2/\mathbb{Z}_2$ orbifold. Let us first start with the bosonic part of the world-sheet action,

$$S_B = -\frac{1}{4\pi\alpha'}\int_\Sigma g_{MN}\partial_a X^M \partial^a X^N + \frac{1}{2}\int_{\partial\Sigma} B_{MN} X^M \partial_\tau X^N \,. \tag{A.1}$$

The boundary condition for the world-sheet bosons from the above action is,

$$g_{MN}\partial_\sigma X^N + 2\pi\alpha' B_{MN}\partial_\tau X^N = 0 \,|_{\sigma=0,\pi} \,. \tag{A.2}$$

In the Seiberg-Witten limit, $g_{ij} = \epsilon\eta_{ij}$ we choose the B field along the brane to be of the form,

$$B = \frac{\epsilon}{2\pi\alpha'}\begin{pmatrix} 0 & b_1 & 0 & 0 \\ -b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_2 \\ 0 & 0 & -b_2 & 0 \end{pmatrix} \tag{A.3}$$

With the above form for the B-field, and defining,

$$X^{\pm}_{(1)} = 2^{-1/2}(X^0 \pm X^1) \qquad \text{and} \qquad X^{\pm}_{(2)} = 2^{-1/2}(X^2 \pm iX^3) \tag{A.4}$$

the boundary condition (A.2) can be rewritten as,

$$\partial_\sigma X^{\pm}_{(1)} = \pm b_1 \partial_\tau X^{\pm}_{(1)} \,|_{\sigma=0,\pi} \qquad \text{and} \qquad \partial_\sigma X^{\pm}_{(2)} = \pm i b_2 \partial_\tau X^{\pm}_{(1)} \,|_{\sigma=0,\pi} \,. \tag{A.5}$$

The mode expansions for the open string satisfying the above boundary conditions are given by,

$$
\begin{aligned}
X^{\pm}_{(1)} &= x^{\pm}_{(1)} + \frac{2\alpha'}{1-b_1^2}(\tau \pm b_1\sigma)p^{\pm}_{(1)} + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{a^{\pm}_{(1)n}}{n}e^{-i(n\tau\pm\nu_1)}\cos(n\sigma\mp\nu_1)\\
X^{\pm}_{(2)} &= x^{\pm}_{(2)} + \frac{2\alpha'}{1+b_2^2}(\tau \pm ib_2\sigma)p^{\pm}_{(2)} + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{a^{\pm}_{(2)n}}{n}e^{-i(n\tau\pm\nu_2)}\cos(n\sigma\mp\nu_2)\,,
\end{aligned}
\tag{A.6}
$$

where we have defined,

$$
i\nu_1 = \frac{1}{2}\log\left(\frac{1+b_1}{1-b_1}\right) \qquad i\nu_2 = \frac{1}{2}\log\left(\frac{1+ib_2}{1-ib_2}\right). \tag{A.7}
$$

The coefficients of the mode expansions (A.6) are fixed so as to satisfy,

$$
\left[X^{+}_{(1)}(\tau,\sigma), P^{-}_{(1)}(\tau,\sigma')\right] = -2\pi\alpha'\delta(\sigma-\sigma') \tag{A.8}
$$

and that the zero modes and the other oscillators satisfy the usual commutation relations,

$$
\left[a^{+}_{(1)m}, a^{-}_{(1)n}\right] = -m\delta_{m+n} \qquad \left[a^{+}_{(2)m}, a^{-}_{(2)n}\right] = m\delta_{m+n} \tag{A.9}
$$

$$
\left[x^{+}_{(1)}, p^{-}_{(1)}\right] = -i \qquad \left[x^{+}_{(2)}, p^{-}_{(2)}\right] = i \tag{A.10}
$$

There is no shift in the moding of the oscillators, the zero point energy and the spectrum is the same as the $B=0$ case. The situation is the same as that of a neutral string in electromagnetic background [22]. Note that the commutator for $X^{\pm}$ now does not vanish at the boundary, for example,

$$
\begin{aligned}
\left[X^{+}_{(1)}(\tau,0), X^{-}_{(1)}(\tau,0)\right] &= -2\pi i\alpha'\frac{b_1}{1-b_1^2}\\
\left[X^{+}_{(1)}(\tau,\pi), X^{-}_{(1)}(\tau,\pi)\right] &= 2\pi i\alpha'\frac{b_1}{1-b_1^2}\,.
\end{aligned}
\tag{A.11}
$$

The zero mode for the energy momentum tensor can now be worked out and is given by,

$$
L^{\parallel}_{(b)0} = \frac{2\alpha'}{b_1^2-1}p^{+}_{(1)}p^{-}_{(1)} + \frac{2\alpha'}{b_2^2+1}p^{+}_{(2)}p^{-}_{(2)} - \sum_{n\neq 0}\left[a^{+}_{(1)-n}a^{-}_{(1)} - a^{+}_{(2)-n}a^{-}_{(2)}\right]. \tag{A.12}
$$

Since the spectrum remains the same, the contribution to the vacuum amplitude from the bosonic modes is the same as the usual $B=0$ case except that there is a factor of $\sqrt{(b_i^2\pm 1)}$ which comes from the trace over the zero modes for each direction along the brane. From (A.3) in the limit (2.5), $b_i \sim 1/\sqrt{\epsilon}$ for B to be finite. With this,

$$
\epsilon^2\prod_i^2 (b_i^2\pm 1) \rightarrow \det(g+2\pi\alpha' B)\,. \tag{A.13}
$$

Including contributions from all the directions,

$$L_{(b)0} = L^{\|}_{(b)0} + L^{\perp}_{(b)0} + L^{orb}_{(b)0} - \frac{5}{12} \tag{A.14}$$

$\perp$ denotes the $4, 5$ directions and $6, 7, 8, 9$ are the orbifolded directions. Let us now compute the contributions from the world sheet fermions. The action is given by,

$$S_F = \frac{i}{4\pi\alpha'} \int_{\Sigma} g_{MN} \bar{\psi}^M \rho^\alpha \partial_\alpha \psi^N - \frac{i}{4} \int_{\partial\Sigma} B_{MN} \bar{\psi}^N \rho^0 \psi^M \,. \tag{A.15}$$

We rewrite the boundary equations from (2.13),

$$g_{MN}(\psi^N_L - \psi^N_R) + 2\pi\alpha' B_{MN}(\psi^N_L + \psi^N_R) = 0 \,|_{\sigma=\pi} \tag{A.16}$$

$$g_{MN}(\psi^M_L + (-1)^a \psi^M_R) + 2\pi\alpha' B_{MN}(\psi^N_L - (-1)^a \psi^N_R) = 0 \,|_{\sigma=0} \;. \tag{A.17}$$

Now defining,

$$\psi^{\pm}_{(1)R,L} = 2^{-1/2}(\psi^0_{R,L} \pm \psi^1_{R,L}) \qquad \text{and} \qquad \psi^{\pm}_{(2)R,L} = 2^{-1/2}(\psi^2_{R,L} \pm i\psi^3_{R,L}) \,. \tag{A.18}$$

For the Ramond Sector ($a = 1$) with the constant B-field given by (A.3),

$$\psi^{\pm}_{(1)R}(1 \pm b_1) = \psi^{\pm}_{(1)L}(1 \mp b_1) \,|_{\sigma=0,\pi} \;. \tag{A.19}$$

Mode expansion,

$$\psi^{\pm}_{(1)L,R} = \sum_n d^{\pm}_{(1)n} \chi^{\pm}_{(1)L,R}(\sigma, \tau, n) \,, \tag{A.20}$$

where,

$$\chi^{\pm}_{(1)R} = \sqrt{2\alpha'} \exp\{-in(\tau - \sigma) \mp \nu_1\} \tag{A.21}$$

$$\chi^{\pm}_{(1)L} = \sqrt{2\alpha'} \exp\{-in(\tau + \sigma) \pm \nu_1\} \tag{A.22}$$

and

$$\nu_1 = \frac{1}{2} \log\left(\frac{1 + b_1}{1 - b_1}\right) = \tanh^{-1} b_1 \,. \tag{A.23}$$

The boundary condition for the other two directions are,

$$\psi^{\pm}_{(2)R}(1 \pm ib_2) = \psi^{\pm}_{(2)L}(1 \mp ib_2) \,|_{\sigma=0,\pi} \;. \tag{A.24}$$

This gives the same mode expansion as (A.20),

$$\psi^{\pm}_{(2)L,R} = \sum_n d^{\pm}_{(2)n} \chi^{\pm}_{(1)L,R}(\sigma, \tau, n) \tag{A.25}$$

$$\chi^{\pm}_{(2)R} = \sqrt{2\alpha'} \exp\{-in(\tau - \sigma) \mp \nu_2\} \tag{A.26}$$

$$\chi^{\pm}_{(2)L} = \sqrt{2\alpha'} \exp\{-in(\tau + \sigma) \pm \nu_2\} \tag{A.27}$$

and

$$\nu_2 = \frac{1}{2} \log\left(\frac{1 + ib_2}{1 - ib_2}\right) = \tan^{-1} b_2 \,. \tag{A.28}$$

Like the bosonic partners there is no shift in the frequencies. The oscillators are integer moded as usual. For the Neveu-Schwarz sector, $(a = 0)$, the relative sign between $\psi_R^\pm$ and $\psi_L^\pm$ at the $\sigma = \pi$ end in eqn(6) can be brought about by the usual restriction on n to only run over half integers in the mode expansions (A.20), (A.25). The oscillators satisfy the standard anticommutation relations,

$$\{d^+_{(1)n}, d^-_{(1)m}\} = -\delta_{m+n}\,; \qquad \{d^+_{(2)n}, d^-_{(2)m}\} = \delta_{m+n}\,. \tag{A.29}$$

The zero mode for the energy momentum tensor for the fermions along the brane can be written as,

$$L^{\|}_{(f)0} = \sum_n n\left[d^-_{(2)-n}d^+_{(2)n} - d^-_{(1)-n}d^+_{(1)n}\right]. \tag{A.30}$$

For all the fermions including the contributions from the other directions we have,

$$L_{(f)0} = L^{\|}_{(f)0} + L^{\perp}_{(f)0} + L^{orb}_{(f)0} + c_f(a)\,, \tag{A.31}$$

where $L^{\perp}_{(f)0}$ and $L^{orb}_{(f)0}$ have the usual representation in terms of oscillators.

$$c_f(1) = \frac{5}{12}\,; \qquad c_f(0) = -\frac{5}{24}\,. \tag{A.32}$$

We now compute the vacuum amplitude including the contributions from the ghosts. This is given by,

$$\mathbb{Z}_C = V_4 \det(g + 2\pi\alpha' B)\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-2}\,\mathrm{Tr}_{NS-R}\left[\left(\frac{1+g}{2}\right)\left(\frac{1+(-1)^F}{2}\right)q^{L_0}\right]. \tag{A.33}$$

The origin of the $\det(g + 2\pi\alpha' B)$ term is given in (A.13) and V_4 is the volume of the D_3-brane and $q = e^{-2\pi t}$. The trace is summed over the spin structures with the orbifold projection. The required traces are listed below in terms of the *Theta Functions*, $\vartheta_i(\nu, it)$ (see for example [23]).

$$Z\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]_e(it) = \mathrm{Tr}_{NS}\left[q^{L_0}\right] = \eta(it)^{-12}\vartheta_3^4(0,it) \tag{A.34}$$

$$\begin{aligned}
Z\left[\begin{smallmatrix}0\\1/2\end{smallmatrix}\right]_e(it) &= \mathrm{Tr}_{NS}\left[(-1)^F q^{L_0}\right] = -\eta(it)^{-12}\vartheta_4^4(0,it)\\
Z\left[\begin{smallmatrix}1/2\\0\end{smallmatrix}\right]_e(it) &= \mathrm{Tr}_{R}\left[q^{L_0}\right] = -\eta(it)^{-12}\vartheta_2^4(0,it)\\
Z\left[\begin{smallmatrix}1/2\\1/2\end{smallmatrix}\right]_e(it) &= \mathrm{Tr}_{R}\left[(-1)^F q^{L_0}\right] = 0\\
Z\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]_g(it) &= \mathrm{Tr}_{NS}\left[g q^{L_0}\right] = 4\eta(it)^{-6}\vartheta_3^2(0,it)\vartheta_4^2(0,it)\vartheta_2^{-2}(0,it)\\
Z\left[\begin{smallmatrix}0\\1/2\end{smallmatrix}\right]_g(it) &= \mathrm{Tr}_{NS}\left[g(-1)^F q^{L_0}\right] = -4\eta(it)^{-6}\vartheta_3^2(0,it)\vartheta_4^2(0,it)\vartheta_2^{-2}(0,it)\\
Z\left[\begin{smallmatrix}1/2\\0\end{smallmatrix}\right]_g(it) &= \mathrm{Tr}_{R}\left[g q^{L_0}\right] = 0\\
Z\left[\begin{smallmatrix}1/2\\1/2\end{smallmatrix}\right]_g(it) &= \mathrm{Tr}_{R}\left[g(-1)^F q^{L_0}\right] = 0\,.
\end{aligned} \tag{A.35}$$

Recalling,

$$\vartheta_3^4(0,it) - \vartheta_4^4(0,it) - \vartheta_2^4(0,it) = 0 \tag{A.36}$$

and noting that,

$$Z\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]_g(it) = -Z\left[\begin{smallmatrix}0\\1/2\end{smallmatrix}\right]_g(it) \tag{A.37}$$

the vacuum amplitude vanishes. This is as a result of supersymmetry.

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